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Rephase-Invariant CP-Violating Observables and Mixings in the B^0-, D^0- and K^0- Systems *

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Abstract

We present a general model-independent and rephase-invariant formalism that cleanly relates observables to the fundamental parameters and explicitly separates different types of CP violation in the B^0-, D^0- and K^0- systems. We emphasize its importance when interpreting experimental measurement of CP violation, the unitarity triangle, and probes of new physics.

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CP violation in neutral meson decays arises from CP violating phases in the mixing matrix (indirect CP violation) or in the weak decay amplitudes (direct CP violation). In the Cabbibo, Kobayashi and Maskawa (CKM) [1] model of CP violation, both direct and indirect CP violation occur. They can be measured by studying time evolution of neutral meson decays [2–6]. Recently there has been much activity connected with tests of this model, particularly on measuring the unitarity triangle through the time evolution measurements [7–9] and time-independent measurements of decay rates by using SU(3) relations for B-meson Decay -amplitudes [10,11]. Other models for CP violation, like the superweak theory [12], or the most general two higgs doublet models [13], may have different predictions for the direct and indirect phases. In the K-system, several experiments are under way to probe direct CP violation and time evolution of kaon decays. In the D-system, mixing and CP violation are expected to be small in the standard model (SM) and thus experimental tests are interesting as a probe of new physics. In the B-system, CP violation could be large in the CKM scheme of the SM, prompting much experimental activity.

Recently, it has been pointed out [14,15] that there are some limitations in extracting the angles of the unitarity triangle by using SU(3) relations for B-meson decay amplitudes as suggested by [11]. Thus it is still necessary to investigate carefully the time-evolution measurements. Basic formula for time-dependent decay rates have been extensively studied [2–9] and applied to various processes. In this note we develop and refine these studies into a general model-independent and rephase-invariant formalism that cleanly relates observables to the fundamental parameters and explicitly separates different forms of CP violation for the neutral meson systems. A meaningful classification of different forms of CP violation must be invariant against phases [16] that can be arbitrarily assigned. In this note we will show that in the neutral meson system there exist in general seven rephase-invariant observables which in principle can be detected by studying the time-evolution of the neutral meson and rate asymmetry. We then conclude that CP-violating observables are in general classified into three types of CP violation and any CP-violating observable can be expressed in terms of seven rephase-invariant quantities. Especially, we emphasize the importance of the rephase-

invariant and model-independent observables when interpreting experimental measurements in the K^0-, D^0- and B^0-systems. Applying this analysis to the decays $B^0 \to \pi^+\pi^-$, $\pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$, we find there are eight physical observables to determine nine input parameters when including the electroweak penguin. Therefore, to extract the angle α of the unitarity triangle, one needs in principle an additional theoretical input. We also examine an interesting decay mode to extract the angle γ.

Let M^0 be the neutral meson (which can be K^0 or D^0 or B^0) and $\bar{M}^0$ its antiparticle. M^0 and $\bar{M}^0$ can mix with each other and form two physical mass eigenstates

$$M_1 = p|M^0> + q|\bar{M}^0>, \qquad M_2 = p|M^0> - q|\bar{M}^0> \tag{1}$$

The CP-violating parameter ϵ_M is introduced via

$$\epsilon_M = \frac{1-q/p}{1+q/p}\ , \qquad \frac{q}{p} \equiv \sqrt{\frac{H_{21}}{H_{12}}} \tag{2}$$

where $H_{12} \equiv M_{12} - \frac{i}{2}\Gamma_{12} =< M^0|H_{eff}|\bar{M}^0 >$.

Let f denote the final decay state of the neutral meson and $\bar{f}$ its charge conjugate state. The decay amplitudes of M^0 and $\bar{M}^0$ are denoted by

$$g \equiv < f|H_{eff}|M^0 >,\ h \equiv < f|H_{eff}|\bar{M}^0 >;\ \bar{g} \equiv < \bar{f}|H_{eff}|\bar{M}^0 >,\ \bar{h} \equiv < \bar{f}|H_{eff}|M^0 > \tag{3}$$

Parameters containing direct CP violation are defined by

$$\epsilon'_M \equiv \frac{1-h/g}{1+h/g},\ \bar{\epsilon}'_M \equiv \frac{1-\bar{g}/\bar{h}}{1+\bar{g}/\bar{h}};\ \epsilon''_M \equiv \frac{1-\bar{g}/g}{1+\bar{g}/g},\ \bar{\epsilon}''_M \equiv \frac{1-h/\bar{h}}{1+h/\bar{h}} \tag{4}$$

Note that the above parameters are not physical observables since they are not rephase-invariant. Let us introduce CP-violating observables by considering the ratio, $\eta_f \equiv < f|H_{eff}|M_2 > / < f|H_{eff}|M_1 >= (1-r_f)/(1+r_f)$, with $r_f = (q/p)(h/g)$ being rephase-invariant. Using a simple algebra relation $1-ab = [(1+a)(1-b)+(1-a)(1+b)]/2$ and $1+ab = [(1+a)(1+b)+(1-a)(1-b)]/2$, it is not difficult to show that η_f can be rewritten as

$$\eta_f = \frac{a_\epsilon + a_{\epsilon'} + i\ a_{\epsilon+\epsilon'}}{2 + a_\epsilon a_{\epsilon'} + a_{\epsilon\epsilon'}} \tag{5}$$

with

$$a_\epsilon = \frac{1-|q/p|^2}{1+|q/p|^2} = \frac{2Re\epsilon_M}{1+|\epsilon_M|^2}\ , \qquad a_{\epsilon'} = \frac{1-|h/g|^2}{1+|h/g|^2} = \frac{2Re\epsilon'_M}{1+|\epsilon'_M|^2}\ ;$$

$$a_{\epsilon+\epsilon'} = \frac{-4Im(qh/pg)}{(1+|q/p|^2)(1+|h/g|^2)} = \frac{2Im\epsilon_M(1-|\epsilon'_M|^2)+2Im\epsilon'_M(1-|\epsilon_M|^2)}{(1+|\epsilon_M|^2)(1+|\epsilon'_M|^2)} \tag{6}$$

$$a_{\epsilon\epsilon'} = \frac{4Re(qh/pg)}{(1+|q/p|^2)(1+|h/g|^2)} - 1 = \frac{4Im\epsilon_M\ Im\epsilon'_M - 2(|\epsilon_M|^2+|\epsilon'_M|^2)}{(1+|\epsilon_M|^2)(1+|\epsilon'_M|^2)}$$

Obviously, a_ϵ, $a_{\epsilon'}$, $a_{\epsilon+\epsilon'}$ and $a_{\epsilon\epsilon'}$ are all rephase-invariant. But only three of them are independent as $(1-a_\epsilon^2)(1-a_{\epsilon'}^2) = a_{\epsilon+\epsilon'}^2 + (1+a_{\epsilon\epsilon'})^2$. Analogously, one has

$$\eta_{\bar{f}} \equiv \frac{<\bar{f}|H_{eff}|M_2>}{<\bar{f}|H_{eff}|M_1>} = \frac{a_\epsilon + a_{\bar{\epsilon}'} + i\ a_{\epsilon+\bar{\epsilon}'}}{2+a_\epsilon a_{\bar{\epsilon}'} + a_{\epsilon\bar{\epsilon}'}} \tag{7}$$

where $a_{\bar{\epsilon}'}$, $a_{\epsilon+\bar{\epsilon}'}$ and $a_{\epsilon\bar{\epsilon}'}$ are similar to $a_{\epsilon'}$, $a_{\epsilon+\epsilon'}$ and $a_{\epsilon\epsilon'}$ but with ϵ'_M being replaced by $\bar{\epsilon}'_M$.

In most phase conventions of the CKM matrix in the literature [1,17], one has, $|\epsilon_K| \ll 1$, for the K-system. In the phase convention of Wu and Yang [18], one has, further, $|\epsilon'_K| \ll 1$. With the fact that $\omega = |A_2/A_0| \ll 1$, one obtains from eq.(6) that $a_\epsilon \simeq 2Re\epsilon_K$, $a_{\epsilon'} \simeq 2Re\epsilon'_K$, $a_{\epsilon+\epsilon'} \simeq 2Im\epsilon_K + 2Im\epsilon'_K$ and $a_{\epsilon\epsilon'} \simeq 0$, and thus

$$\eta_{+-} \simeq Re\epsilon_K + Re\epsilon'_K + i(Im\epsilon_K + Im\epsilon'_K) = \epsilon_K + \epsilon'_K \tag{8}$$

which reproduces the form often used in the literature for $K^0 \to \pi^+\pi^-$ decay.

Two additional rephase-invariant quantities complete the set of observables,

$$a_{\epsilon''} = \frac{1-|\bar{g}/g|^2}{1+|\bar{g}/g|^2} = \frac{2Re\epsilon''_M}{1+|\epsilon''_M|^2}, \qquad a_{\bar{\epsilon}''_M} = \frac{1-|\bar{h}/h|^2}{1+|\bar{h}/h|^2} = \frac{2Re\bar{\epsilon}''_M}{1+|\bar{\epsilon}''_M|^2} \tag{9}$$

So far we have introduced five parameters from which seven independent rephase-invariant observables are constructed to describe CP violation: ϵ_M is an indirect CP-violating parameter; ϵ''_M and $\bar{\epsilon}''_M$ define direct CP-violating parameters; ϵ'_M and $\bar{\epsilon}'_M$ contain the ratio of the two decay amplitudes and can be associated with direct CP violation as well as the interference between indirect and direct CP violation. All the CP violations can be well defined and in general classified into the following three types: i) purely indirect CP violation which is given by the rephase-invariant CP-violating observable a_ϵ; ii) purely direct CP violation

which is characterized by the rephase-invariant CP-violating observables $a_{\epsilon''}$ and $a_{\bar{\epsilon}''}$; and iii) indirect-direct mixed CP violation which is described by the rephase-invariant CP-violating observables $a_{\epsilon+\epsilon'}$ and $a_{\epsilon+\bar{\epsilon}'}$. For the case that the final states are CP eigenstates, one has $a_{\epsilon'} = a_{\epsilon''} = a_{\bar{\epsilon}'} = a_{\bar{\epsilon}''}$. Thus, in this case $a_{\epsilon'}$ and $a_{\bar{\epsilon}'}$ also indicate purely direct CP violation. When the final states are not CP eigenstates, $a_{\epsilon'}$ and $a_{\bar{\epsilon}'}$ do not, in general, provide a clear signal of direct CP violation although they contain direct CP violation. Their deviation from $a_{\epsilon'} = \pm 1,\ 0$ and $a_{\bar{\epsilon}'} = \mp 1,\ 0$ can arise from different CKM angles, final state interactions, or different hadronic form factors, but not necessarily from CP violation.

In order to measure these rephase-invariant observables, we consider the proper time evolution of the neutral mesons

$$|M^0(t)>= \sum_{i=1}^{2} C_i e^{-i(m_i - i\Gamma_i/2)t}|M_i> ; \qquad |\bar{M}^0(t)>= \sum_{i=1}^{2} \bar{C}_i e^{-i(m_i - i\Gamma_i/2)t}|M_i> \tag{10}$$

with $C_1 = C_2 = 1/2p$ and $\bar{C}_1 = -\bar{C}_2 = 1/2q$ for purely M^0 and $\bar{M}^0$ at $t = 0$. The time-dependent decay rates are found to be

$$\Gamma(M^0(t) \to f) \propto |<f|H_{eff}|M^0(t)>|^2 = \frac{1}{1+a_\epsilon}\frac{(|g|^2+|h|^2)}{2}e^{-\Gamma t} \tag{11}$$
$$\cdot[(1+a_\epsilon a_{\epsilon'})\cosh(\Delta\Gamma t) + (1+a_{\epsilon\epsilon'})\sinh(\Delta\Gamma t) + (a_\epsilon + a_{\epsilon'})\cos(\Delta m t) + a_{\epsilon+\epsilon'}\sin(\Delta m t)]$$
$$\Gamma(\bar{M}^0(t) \to \bar{f}) \propto |<\bar{f}|H_{eff}|\bar{M}^0(t)>|^2 = \frac{1}{1-a_\epsilon}\frac{(|\bar{g}|^2+|\bar{h}|^2)}{2}e^{-\Gamma t} \tag{12}$$
$$\cdot[(1+a_\epsilon a_{\bar{\epsilon}'})\cosh(\Delta\Gamma t) + (1+a_{\epsilon\bar{\epsilon}'})\sinh(\Delta\Gamma t) - (a_\epsilon + a_{\bar{\epsilon}'})\cos(\Delta m t) - a_{\epsilon+\bar{\epsilon}'}\sin(\Delta m t)\ ]$$

One can easily write down the decay rates $\Gamma(M^0(t) \to \bar{f})$ and $\Gamma(\overline{M}^0(t) \to f)$. where $\Delta\Gamma = \Gamma_2 - \Gamma_1$ and $\Delta m = m_2 - m_1$. Here we have omitted the integral of the phase space.

From studying the time-dependent spectrum of the decay rates of M^0 and $\bar{M}^0$, one can, in principle, find the coefficients of the four functions $\sinh(\Delta\Gamma t)$, $\cosh(\Delta\Gamma t)$, $\cos(\Delta m t)$ and $\sin(\Delta m t)$ and extract the quantities a_ϵ, $a_{\epsilon'}$, $a_{\epsilon+\epsilon'}$, $|g|^2+|h|^2$, $a_{\bar{\epsilon}'}$, $a_{\epsilon+\bar{\epsilon}'}$ and $|\bar{g}|^2+|\bar{h}|^2$ as well as Δm, $\Delta\Gamma$ and Γ. Thus, one can determine the amplitudes $|q/p|$, $|g|^2$, $|h|^2$, $|\bar{g}|^2$, $|\bar{h}|^2$ and combinations of the phases $(\phi_M + \phi_A)$ as well as $(\phi_M + \bar{\phi}_A)$ * via

* Note that only the combination of the two phases is rephase-invariant.

$$|\frac{q}{p}|^2 = \frac{1-a_\epsilon}{1+a_\epsilon}, \qquad |\frac{h}{g}|^2 = \frac{1-a_{\epsilon'}}{1+a_{\epsilon'}}, \qquad \sin(2(\phi_M+\phi_A)) = \frac{a_{\epsilon+\epsilon'}}{\sqrt{(1-a_\epsilon^2)(1-a_{\epsilon'}^2)}} \tag{13}$$

where $q/p = |q/p|e^{-2i\phi_M}$, $h/g = |h/g|e^{-2i\phi_A}$ and $\bar{g}/\bar{h} = |\bar{g}/\bar{h}|e^{-2i\bar{\phi}_A}$. For $f^{CP} = \pm f$, one has $\phi_A = \bar{\phi}_A$. For $f \neq f^{CP}$, $\phi_A = \bar{\phi}_A$ holds only when final state interactions are absent, so that $\phi_A \neq \bar{\phi}_A$ implies an existence of final state interactions when f is not a CP eigenstate.

The time-dependent CP asymmetry is the difference between the two decay rates of eqs. (11) and (12). In terms of the rephase-invariant quantities, we have

$$\begin{aligned}
\Delta_{CP}(t) = \Gamma(M^0(t)\to f) - \Gamma(\overline{M}^0(t)\to \bar{f}) = {} & \frac{1}{1-a_\epsilon^2}\frac{1}{2}e^{-\Gamma t}\frac{(|\bar{g}|^2+|g|^2)}{2}\cdot \\
& \{[(1+a_{\epsilon''})(1-a_\epsilon)(a_\epsilon+a_{\epsilon'}) + (1-a_{\epsilon''})(1+a_\epsilon)(a_\epsilon+a_{\bar{\epsilon}'})]\cos(\Delta m t) \\
& +[(1+a_{\epsilon''})(1-a_\epsilon)a_{\epsilon+\epsilon'} + (1-a_{\epsilon''})(1+a_\epsilon)a_{\epsilon+\bar{\epsilon}'}]\sin(\Delta m t) \\
& -[(a_\epsilon-a_{\epsilon''})(2+(a_{\bar{\epsilon}'}+a_{\epsilon'})a_\epsilon) + (1-a_\epsilon a_{\epsilon''})(a_{\bar{\epsilon}'}-a_{\epsilon'})a_\epsilon]\cosh(\Delta\Gamma t) \\
& -[(a_\epsilon-a_{\epsilon''})(2+a_{\epsilon\bar{\epsilon}'}+a_{\epsilon\epsilon'}) + (1-a_\epsilon a_{\epsilon''})(a_{\epsilon\bar{\epsilon}'}-a_{\epsilon\epsilon'})]\sinh(\Delta\Gamma t)\} \\
& +(g\leftrightarrow h\ ,\bar{g}\leftrightarrow \bar{h}\ ,\ a_{\epsilon''}\leftrightarrow -a_{\bar{\epsilon}''})
\end{aligned} \tag{14}$$

One can in general define several asymmetries from the four time-dependent decay rates $\Gamma(M^0(t)\to f)$, $\Gamma(\overline{M}^0(t)\to\bar{f})$, $\Gamma(M^0(t)\to\bar{f})$ and $\Gamma(\overline{M}^0(t)\to f)$.

To apply the above general analyses to specific processes, we may classify the processes into the following scenarios

i) $M^0\to f$ ($M^0 \not\to \bar{f}$) , $\overline{M}^0 \to \bar{f}$ ($\overline{M}^0 \not\to f$) , i.e., f or $\bar{f}$ is not a common final state of M^0 and $\overline{M}^0$. Examples are: $M^0\to M'^-\bar{l}\nu$, $\bar{M}^0\to M'^+ l\bar{\nu}$; $B^0\to D^-D_s^+$; $\bar{B}^0\to D^+D_s^-$. , This scenario also applies to charged meson decays [19].

ii) $M^0 \to (f=\bar{f},\ f^{CP}=f) \leftarrow \overline{M}^0$, i.e., final states are CP eigenstates. Such as $B^0(\bar{B}^0)$, $D^0(\bar{D}^0)$, $K^0(\bar{K}^0)\to\pi^+\pi^-$, $\pi^0\pi^0$, $\cdots$. For the final states such as $\pi^-\rho^+$ and $\pi^+\rho^-$, although each of them is not a CP eigenstate of $B^0(\bar{B}^0)$ or $D^0(\bar{D}^0)$, one can always reconstruct them into CP eigenstates as $(\pi\rho)_\pm = (\pi^-\rho^+ \pm \pi^+\rho^-)$ with $CP(\pi\rho)_\pm = \pm(\pi\rho)_\pm$. This reconstruction is meaningful since $\pi^-\rho^+$ and $\pi^+\rho^-$ have the same weak phase as they contain the same quark content.

iii) $M^0 \to (f,\ f \not\to f^{CP}) \leftarrow \overline{M}^0$, i.e., final states are common final states but they are not charge conjugate states. For example, $B^0(\bar{B}^0) \to K_S J/\psi$, $B_s^0(\bar{B}_s^0) \to K_S \phi$.

iv) $M^0 \to (f \ \&\ \bar{f},\ f^{CP} \neq f) \leftarrow \overline{M}^0$, i.e., both f and $\bar{f}$ are the common final states of M^0 and $\overline{M}^0$, but they are not CP eigenstates. This is the most general case. For example, $B^0(\bar{B}^0) \to D^-\pi^+,\ \pi^- D^+$; $D^-\rho^+,\ \rho^- D^+$; $B_s^0(\bar{B}_s^0) \to D_s^- K^+,\ K^- D_s^+$.

In the scenario i), one has: $a_{\epsilon'} = -a_{\bar{\epsilon}'} = 1$, $a_{\epsilon+\epsilon'} = 0 = a_{\epsilon+\bar{\epsilon}'}$ and $a_{\epsilon\epsilon'} = -1 = a_{\epsilon\bar{\epsilon}'}$. The time-dependent rates of eqs. (11) and (12) then become much simpler. Thus, Δm, $\Delta\Gamma$, a_ϵ and $a_{\epsilon''}$ can be easily extracted via

$$A_{CP}(t) = \Delta_{CP}(t)/(\Gamma(M^0(t) \to f) + \Gamma(\overline{M}^0(t) \to \bar{f})) = a_{\epsilon''} \tag{15}$$

$$A'_{CP}(t) = \frac{\Gamma(\overline{M}^0(t) \to f) - \Gamma(M^0(t) \to \bar{f})}{\Gamma(\overline{M}^0(t) \to f) + \Gamma(M^0(t) \to \bar{f})} = (a_{\epsilon''} + \frac{2a_\epsilon}{1+a_\epsilon^2})/(1 + \frac{2a_\epsilon}{1+a_\epsilon^2} a_{\epsilon''}) \tag{16}$$

$$\frac{\cos(\Delta m t)}{\cosh(\Delta\Gamma t)} = \frac{A^f(t) + a_\epsilon}{1 + A^f(t) a_\epsilon}; \qquad A^f(t) = \frac{\Gamma(M^0(t) \to f) - \Gamma(\overline{M}^0(t) \to f)}{\Gamma(M^0(t) \to f) + \Gamma(\overline{M}^0(t) \to f)}. \tag{17}$$

It is interesting to note that the CP asymmetries A_{CP} and A'_{CP} are actually time-independent since the time- dependent parts cancel in the ratio. Indirect CP violation a_ϵ in the B^0- and B_s^0-systems may be directly obtained by measuring the decay channels in the scenario i), such as $B^0 \to D^- K^+$, $\pi^- D_s^+$ and $B_s^0 \to D_s^- \pi^+$, $K^- D^+$, respectively, or from their semileptonic decays, since in these decays one has $a_{\epsilon''} = 0$. Purely direct CP violation $a_{\epsilon''}$ in the B^0- and B_s^0-systems may be detected by studying decay modes such as $B^0 \to D^- D_s^+$, $\pi^- K^+$, and $B_s^0 \to D_s^- D^+$, $K^-\pi^+$. These decay modes receive contributions from both tree and penguin diagrams so that the final state interactions may become significant.

We now discuss the scenario ii) in which $a_{\epsilon'} = a_{\epsilon''} = a_{\bar{\epsilon}'} = a_{\bar{\epsilon}''}$ and $a_{\epsilon+\epsilon'} = a_{\epsilon+\bar{\epsilon}'}$. Thus, the time-dependent CP asymmetry simplifies to

$$\begin{aligned} A_{CP}(t) &= (\Delta_m(t) - a_\epsilon\ \Delta_\gamma(t))/(\Delta_\gamma(t) - a_\epsilon\ \Delta_m(t)) \\ \Delta_m(t) &= (a_\epsilon + a_{\epsilon'}) \cos(\Delta m t) + a_{\epsilon+\epsilon'} \sin(\Delta m t) \\ \Delta_\gamma(t) &= (1 + a_\epsilon a_{\epsilon'}) \cosh(\Delta\Gamma t) + (1 + a_{\epsilon\epsilon'}) \sinh(\Delta\Gamma t) \end{aligned} \tag{18}$$

Suppose that Δm, $\Delta\Gamma$ and a_ϵ are known from studying the processes in the scenario

i); one then can extract the direct CP-violating observable $a_{\epsilon'}$ and CP-violating observable $a_{\epsilon+\epsilon'}$ from the coefficients of $\cos(\Delta mt)$ and $\sin(\Delta mt)$ respectively from type (ii) processes.

Let us consider the following special but realistic cases:

1) $a_\epsilon \ll 1$, then, to the first order of a_ϵ and $a_{\epsilon'}$, one has

$$A_{CP}(t) \simeq -a_\epsilon + \frac{(a_\epsilon + a_{\epsilon'})\cos(\Delta mt) + a_{\epsilon+\epsilon'}\sin(\Delta mt)}{\cosh(\Delta\Gamma t) + (1 + a_{\epsilon\epsilon'})\sinh(\Delta\Gamma t)} \tag{19}$$

This case actually holds for all the neutral meson systems. In the $K^0 - \bar{K}^0$ system, one also has $a_{\epsilon'} \ll 1$ and $\Delta m \simeq -\Delta\Gamma/2$.

2) $a_\epsilon \ll 1$, $|\Delta\Gamma| \ll |\Delta m|$ and $|\Delta\Gamma/\Gamma| \ll 1$. Then, $A_{CP}(t)$ further simplifies

$$A_{CP}(t) \simeq -a_\epsilon + (a_\epsilon + a_{\epsilon'})\cos(\Delta mt) + a_{\epsilon+\epsilon'}\sin(\Delta mt) \tag{20}$$

which may be applied, in a good approximation, to the $B^0 - \bar{B}^0$ system.

As an interesting example, let us reanalyze the $\pi\pi$ processes in the context of our formalism and show how eight observables may be extracted from the data.

The decay amplitudes of $B^0 \to \pi^+\pi^-$, $\pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ can be expanded, in terms of the isospin ($I = 0, 2$) and the tree- and penguin-diagrams, into

$$g_{+-} = \sqrt{2}(A_{0T}e^{-i\phi_T + i\delta_{0T}} + A_{0P}e^{-i\phi_P + i\delta_{0P}} - A_{2T}e^{-i\phi_T + i\delta_{2T}} - A_{2P}e^{-i\phi_P + i\delta_{2P}}) \tag{21}$$

$$g_{00} = A_{0T}e^{-i\phi_T + i\delta_{0T}} + A_{0P}e^{-i\phi_P + i\delta_{0P}} + 2(A_{2T}e^{-i\phi_T + i\delta_{2T}} + A_{2P}e^{-i\phi_P + i\delta_{2P}}) \tag{22}$$

$$g_{+0} = 3(A_{2T}e^{-i\phi_T + i\delta_{2T}} + A_{2P}e^{-i\phi_P + i\delta_{2P}}) \tag{23}$$

where A_{0i} and A_{2i} are the isospin $I = 0$ and 2 amplitudes of the tree diagram ($i = T$) and penguin diagrams ($i = P$), δ_{0i} and δ_{2i} ($i = T,\ P$) are the corresponding strong phases. Where A_{2P} arises from the electroweak penguin. ϕ_T and ϕ_P are the weak phases of the tree- and penguin- diagrams respectively. Thus, we have nine physical quantities. They consist of five phases and four amplitudes:

$$\begin{aligned} &\Delta\phi = \phi_T - \phi_P,\ (\phi_M + \phi_T)\ or\ (\phi_M + \phi_P),\ \Delta\delta_T = \delta_{0T} - \delta_{2T}, \\ &\Delta\delta_P = \delta_{0P} - \delta_{2T},\ \Delta\delta'_P = \delta_{2P} - \delta_{2T},\ A_{0T},\ A_{2T},\ A_{0P},\ A_{2P} \end{aligned} \tag{24}$$

while we have in general eight independent observables, they are:

$$a_{\epsilon'}^{(+-)},\ a_{\epsilon'}^{(00)},\ \ a_{\epsilon''}^{(+0)},\ a_{\epsilon+\epsilon'}^{(+-)},\ a_{\epsilon+\epsilon'}^{(00)},\ <\Gamma_{+-}>,\ <\Gamma_{00}>,\ <\Gamma_{+0}> \tag{25}$$

which implies that without using a theoretical input one cannot in principle unambiguously extract the angle α from $B \to \pi\pi$. A model calculation however shows that $A_{2p}/A_{2T} \simeq 1.6\%(|V_{td}|/|V_{ub}|)$ [14]. If the contribution of the electroweak penguin is negligible, it is remarkable that there are only seven quantities (omitting $\Delta\delta'_P$ and A_{2P} in eq. (24)) which can be extracted from seven observables (omitting $a_{\epsilon''}^{(+0)}$ in eq. (25)), where $a_{\epsilon'}^{(+-)}$ and $a_{\epsilon'}^{(00)}$ are proportional to $\sin(\Delta\delta_P)\sin(\Delta\phi)$ and $\sin(\Delta\delta_T - \Delta\delta_P)\sin(\Delta\phi)$; $a_{\epsilon+\epsilon'}^{(+-)}$ and $a_{\epsilon+\epsilon'}^{(00)}$ are related to combinations of $\sin(\phi_M + \phi_T)$, $\cos(\Delta\delta_P)$, $\cos(\Delta\delta_T)$ and the relative amplitude of the tree and penguin diagrams. Together with the decay rates Γ_{+-}, Γ_{00} and Γ_{+0}, one can in principle extract the seven quantities in eq. (24) as also noted by [7]. In the CKM scheme, one has $\phi_M = \beta$, $\phi_T = \gamma$ and $\phi_P \simeq -\beta$, thus

$$(\phi_M + \phi_T) = \beta + \gamma = \pi - \alpha, \qquad \Delta\phi = \phi_T - \phi_P \simeq \beta + \gamma = \pi - \alpha \tag{26}$$

A model for the strong phase of the strong penguin diagram has recently been studied by [19,20]. As an illustration of the effect of strong phase and amplitude of the strong penguin diagram, we present in the table 1 the values of $a_{\epsilon'}^{(+-)}$, $a_{\epsilon'}^{(00)}$, $a_{\epsilon+\epsilon'}^{(+-)}$, and $a_{\epsilon+\epsilon'}^{(00)}$ with $a_\epsilon = 0$ in the decays $B^0 \to \pi^+\pi^-$, $\pi^0\pi^0$ as functions of representative Wolfenstein parameters ρ and η of the CKM matrix in a BSW model [21] using the methods and parameters of refs. [19,22]. As we see from table 1, the influence of the amplitude and strong phase of the penguin diagram is sizable even in the $B^0 \to \pi^+\pi^-$ channel but is quite drastic in the $B^0 \to \pi^0\pi^0$ channel, where color suppression occurs for the tree amplitude. It should also be noted how sensitive $a_{\epsilon+\epsilon'}^{(00)}$ is to the CKM parameters, changing sign between the two prefered Ali and London [22] solutions due to the cancellation between tree and strong penguin contributions. For certain values of ρ and η, $a_{\epsilon+\epsilon'}^{(00)} \simeq 0$. In this case the electroweak penguin could be a complication in the $\pi^0\pi^0$ channel when extracting the angle α. The effect of the electroweak penguin diagram may be seen by studying the CP asymmetry $a_{\epsilon''}^{(+0)}$ in the charged B-meson decay.

A similar analysis can be applied to the K- and D-systems. It is of interest that for the K-system it becomes simpler, i.e. $\delta_{IT} = \delta_{IP} = \delta_I$ ($I = 0{,}2$). This is because in the K-system $\pi\pi$ scattering is assumed to be purely elastic (in the good approximation of neglecting the $\pi\pi\gamma$ final state), while in the B-system $\pi\pi$ scattering can be inelastic. Thus in the K-system there are only seven parameters in eq. (24) instead of nine, but there are also only six independent observables in eq. (25) because $\Delta = [\Gamma(\pi^+\pi^-) - \bar{\Gamma}(\pi^+\pi^-)] + [\Gamma(\pi^0\pi^0) - \bar{\Gamma}(\pi^0\pi^0)] = 0$ as required by CPT in the absence of other channels. In the B-system there are two more parameters as well as two more observables as shown above and $\Delta \neq 0$. Therefore the measurement of Δ distinguishes CP violation in the K-system from that of B-system †.

We now present another interesting decay mode to extract the angle γ, i.e., $B_s^0 \to (D_s^- K^+,\ K^- D_s^+) \leftarrow \bar{B}_s^0$. Since these decay modes only receive contributions from tree diagram, the phase ϕ_A of the amplitude is almost a purely weak phase and given by $2\phi_A = \gamma$ in the CKM scheme. The phase ϕ_M from the $B_s^0 - \bar{B}_s^0$ mixing is expected to be small in the CKM scheme, $\phi_M \ll 1$. By measuring the rephase-invariant quantities $a_{\epsilon+\epsilon'}$ and $a_{\epsilon'}$ in the above decay modes, one then extracts the angle γ (see equation (13))

$$\sin(2(\phi_M + \phi_A)) \simeq \sin\gamma = \frac{a_{\epsilon+\epsilon'}}{\sqrt{(1-a_\epsilon^2)(1-a_{\epsilon'}^2)}} \simeq \frac{a_{\epsilon+\epsilon'}}{\sqrt{(1-a_{\epsilon'}^2)}} \tag{27}$$

In general, such a triangle is not necessarily closed as long as either the mass mixing matrices or the amplitudes receive contributions from new sources of CP violation or new interactions beyond the standard model, such as the most general two-Higgs doublet model [13].

We hope that the general model-independent and rephase-invariant formalism developed in this note will be useful in the analysis of the neutral meson systems to test the SM and probe new physics.

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Table 1. $B \to \pi\pi$ CP-violating observables in a standard model calculation using the BSW model [21], absorptive parts of the penguins in NLL formulism [20], and the CKM parameters of a recent fit [22].

CP Violation in $B \to \pi\pi$ Tree (T) and Penguins with (P) or without (P') Absorptive Parts NLL QCD Coefficients, BSW model					
	Assumptions		**Decay Parameters**		
Channel	CKM (ρ, η)	Amplitude	$a_{\epsilon'}$	$a_{\epsilon+\epsilon'}$	$< BR >$
$\pi^0\pi^0$	(-0.12,0.34)	T + P	0.316	0.201	4.25×10^{-7}
	”	T + P'	0.0	0.216	4.14×10^{-7}
	”	T	0.0	-0.951	5.66×10^{-7}
$\pi^0\pi^0$	(-0.28,0.24)	T + P	0.496	-0.465	1.91×10^{-7}
	”	T + P'	0.0	-0.487	1.77×10^{-7}
	”	T	0.	-0.869	5.93×10^{-7}
$\pi^+\pi^-$	(-0.12,0.34)	T + P	0.0708	-0.766	1.19×10^{-5}
	”	T + P'	0.	-0.777	1.18×10^{-5}
	”	T	0.0	-0.951	1.43×10^{-5}
$\pi^+\pi^-$	(-0.28,0.24)	T + P	0.0573	-0.967	1.03×10^{-5}
	”	T + P'	0.0	-0.97	1.03×10^{-5}
	”	T	0.0	-0.869	1.50×10^{-5}